Math 21D, Spring 2022 – Midterm 1 Review

Wednesday, April 13

1. Evaluate the integral:

   (a) $\displaystyle\int_0^1 \int_0^{x^3} e^{y/x}\, dy\, dx$

   (b) $\displaystyle\int_0^1 \int_{\sqrt{y}}^{2-\sqrt{y}} xy\, dx\, dy$

   (c) $\displaystyle\int_0^2 \int_{y/2}^1 e^{x^2}\, dx\, dy$

   (d) $\displaystyle\int_0^1 \int_{\sqrt[3]{y}}^1 \frac{2\pi \sin \pi x^2}{x^2}\, dx\, dy$

   (e) $\displaystyle\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2+y^2+1)\, dx\, dy$

   (f) $\displaystyle\int_1^e \int_1^x \int_0^z \frac{2y}{z^3}\, dy\, dz\, dx$

2. Find the volume under the paraboloid $z = x^2+y^2$ above the triangle enclosed by $y = x$, $x = 0$, and $x + y = 2$ in the xy-plane.

3. Find the average value of the function $f(x, y) = xy$ over the quarter circle $x^2 + y^2 = 1$ in the first quadrant.

4. Find the volume of the region enclosed on the top by the plane $z = -2x$, on the side by the cylinder $x = -\cos y$, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, and below by the xy-plane.

5. Find the average value of $f(x, y, z) = 30xz\sqrt{x^2 + y}$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 1$, $y = 3$, $z = 1$.

6. Convert $\displaystyle\int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} 3\, dz\, r\, dr\, d\theta$ to rectangular coordinates in the order $dz\, dx\, dy$ and to spherical coordinates. Then evaluate one of the integrals.

7. Write the integral of $f(x, y, z) = 6 + 4y$ over the region in the first octant bounded by the cone $z = \sqrt{x^2 + y^2}$, the cylinder $x^2+y^2 = 1$, and the coordinate planes in all three coordinate systems and evaluate one of the integrals.

8. Find the volume of the region bounded above by the sphere $x^2+y^2+z^2 = 8$ and below by the plane $z = 2$ using cylindrical and spherical coordinates.

9. Find the centroid of the "triangular" region bounded by the lines $x = 2$ and $y = 2$ and the hyperbola $xy = 2$.

10. Use the substitution $u = x - y$, $v = y$ to show that

$$\int_0^\infty \int_0^x e^{-sx} f(x-y, y)\, dy\, dx = \int_0^\infty \int_0^\infty e^{-s(u+v)} f(u, v)\, du\, dv$$

    if f is any continuous function.