10th Annual Davis Math Conference

Schedule and Abstracts

Organizing committee: Kyle Chickering, Yanwen Luo, and Jingyang Shu

Thursday, January 9th 2020

1 Schedule

Continental breakfast	9:00 AM – 9:25 AM	
Opening remarks	9:25 AM – 9:30 AM	Black Jiang
Math Ecology	9:30 AM – 9:55 AM	Alan Hastings
Geometry and Topology	10:00 AM – 10:25 AM	Maria Trnkova
Algebraic Combinatorics	10:30 AM – 10:55 AM	Tair Akhmejanov
Coffee break	10:55 AM – 11:15 AM	
Analysis of PDEs	11:15 AM – 11:40 AM	Annalaura Stingo
Probability	11:45 AM – 12:10 PM	Daniel Blanquicett
Lunch	12:10 PM – 1:15 PM	
Molecular Computing	1:15 PM – 1:40 PM	David Doty
Optimization	1:45 PM – 2:10 PM	David Woodruff
Coffee break	2:10 PM – 2:30 PM	
Applied Algebraic Geometry	2:30 PM – 2:55 PM	Robert Krone
Math Physics	3:00 PM – 3:25 PM	Sean Curry
Department tea	3:30 PM – 4:30 PM	

*This conference is supported by the Galois Group, the Graduate Student Association, and the Department of Mathematics of UC Davis.

2 Titles and Abstracts

2.1 Math Ecology

Alan Hastings

Title: Transient dynamics in ecology

Abstract: I will present evidence that transients are important for understanding ecological systems and indicate how to begin to understand these using idea from dynamical systems.

2.2 Geometry and Topology

Maria Trnkova

Title: Approximating surfaces by meshes

Abstract: In this talk we will discuss a construction of a mesh that approximates an embedded surface F in R^3, and common problems appearing in this process. There are many different algorithms that aim to construct a good approximation of F with triangles satisfying special properties. We will briefly describe the Marching Cubes algorithm and then introduce a new one, called GradNormal. The idea is to tile the space by a single shape tetrahedra that belong to Goldberg's family of tilings, and to approximate a surface given as a level set by normal sections of a tetrahedral lattice. All triangles in the resulting mesh guaranteed to have angles between 35.2 and 101.5 degrees as the scaling parameter of the tetrahedral lattice goes to zero or in other words as a mesh becomes finer. This is a joint work with Joel Hass.

2.3 Algebraic Combinatorics

Tair Akhmejanov

Title: Kuperberg Spider Categories

Abstract: I will discuss combinatorial representation theory, focusing on Kuperberg's webs and spider categories, which are defined as follows. The representation theory of a Lie algebra or a quantum group forms a pivotal tensor category. Such a category can be studied combinatorially using a diagrammatic presentation by generating morphisms and relations (much like a presentation of a ring or group). Each morphism is a diagram that, when drawn in a disk, resembles a web, and such diagrams can be composed and tensored to generate bigger diagrams. In 1996, Kuperberg gave a set of generating morphisms for each of the rank 2 Lie algebras $\mathfrak{sl}_3, \mathfrak{so}_5, \mathfrak{g}_2$ and a complete set of relations. Morrison did the same for $\mathfrak{sl}_n$ in 2007. This question remains open in other types. Given a presentation, another interesting line of research is to specify a "good" basis of webs for every Hom-space of the tensor category.

2.4 Analysis of PDEs

Annalaura Stingo

Title: Almost global well-posedness for quadratic quasilinear 2D wave-Klein-Gordon systems with small and localized initial data

Abstract: In this talk we will briefly discuss the almost global existence of small solutions of strongly coupled systems of a wave equation and a Klein-Gordon equation (WKG) in $2+1$ space-time dimensions. The coupling we consider is quadratic, quasilinear, and satisfies the so-called null structure. No restriction is made on the support of the initial data, that are small and only mildly decaying at infinity.

Wave-Klein-Gordon systems arise from models strictly related to General Relativity, but only few results are known at present and most of them are proved in $3+1$ space-time dimensions. In $2+1$ space-time dimensions, the only contributions to the subjects are due to Y. Ma, who proves the global-well-posedness for some particular examples of weakly coupled WKG with small compactly supported initial data, and to ourselves, as we prove the global existence of solutions of a model strong coupled WKG with small and localized data (no restriction on their support).

The work we present here, in collaboration with M. Ifrim, addresses a much wider class of strongly coupled WKG. We prove that small solutions are almost global, using a combination of energy estimates localized to dyadic space-time region and pointwise interpolation type estimates within the same regions. This is akin to ideas previously used by Metcalfe-Tataru-Tohaneanu in a linear setting, and is also related to Alinhac's ghost weight method. A refinement of these estimates will lead us to pass, in a future work, from almost global existence to global existence of solutions under the same hypothesis on the initial data.

2.5 Probability

Daniel Blanquicett

Title: Bootstrap percolation

Abstract: Bootstrap percolation is a monotone version of the Glauber dynamics of the Ising model of ferromagnetism. The r-neighbour bootstrap process on a locally finite graph G is a monotone cellular automata on the configuration space $\{0,1\}^{V(G)}$, where the initial state is usually chosen to be the product of Bernoulli measures with density p.

In this talk we will discuss extremal and probabilistic questions in the area, by considering bootstrap processes on $[L]^d$. We will mainly focus on the so-called *critical length for percolation* $L_c(p)$, for small values of p, and challenging open problems will be mentioned.

2.6 Molecular Computing

David Doty

Title: Crystals that think about how they're growing

Abstract: Advances in software engineering and programming languages have enabled vastly complex, yet reliable, electronic computing systems. Imagine a world where smart molecules, controlled by programmable chemical reactions, achieve the same level of precise automated control over the configuration of matter at the molecular level. Theory that combines mathematical tiling and statistical-mechanical models of crystallization has shown that algorithmic behavior can be embedded within molecular self-assembly processes. Previous DNA nanotechnology results had experimentally demonstrated algorithmic "tile" self-assembly with up to 22 tile types, creating patterns such as Sierpinkski triangles and binary counters. Despite that success, many information technologies exhibit a complexity threshold – such as the minimum transistor count needed for a general-purpose computer – beyond which there is a qualitative increase in the power of a reprogrammable system, and it has not been clear whether the biophysics of DNA self-assembly would allow that threshold to be exceeded.

Here we report the design and experimental validation of a DNA tile set containing 355 single-stranded tiles, reprogrammable by tile selection to implement a wide variety of 6-bit algorithms, including copying, sorting, recognizing palindromes and multiples of 3, random walking, obtaining an unbiased choice from a biased random source, electing a leader, simulating cellular automata, generating deterministic and randomized patterns, and serving as a period 63 counter. The system is quite reliable: averaged across the 21 implemented circuits, the per-tile error rate is less than 1 in 3000. Just as high-level programming languages enable apps to be written without knowing anything about transistors, the development of multipurpose molecular machines, reprogrammable without knowledge of the machine's physics, would establish a creative space where high-level molecular programmers can flourish.

2.7 Optimization

David Woodruff

Title: Applied aspects of optimization under uncertainty

Abstract: Optimization problems crop up in all sorts of places and the input data often concern the future and are therefore uncertain. In this talk we will take as given the technology for solving an optimization problem with given input data and concern ourselves with three things: 0) characterizing the uncertainty 1) solving a problem that takes the uncertainty into account, and 2) characterizing uncertainty associated with the solution.

2.8 Applied Algebraic Geometry

Robert Krone

Title: Matrices on the nonnegative rank boundary

Abstract: The nonnegative rank of a matrix is the minimum number of nonnegative rank-1 summands needed to express it, and its computation has applications in statistics and data science. The problem of determining whether the nonnegative rank of a matrix is equal to its rank has a beautiful reformulation in terms of nesting polytopes. In this framework, the case of nonnegative rank 3 is well understood, but the situation for higher ranks is much more wild. We describe some of matrices on the boundary of having nonnegative rank 4 and higher. This is joint work with Kaie Kubjas.

2.9 Math Physics

Sean Curry

Title: Three dimensional Cauchy-Riemann geometry

Abstract: We will introduce the topic of three dimensional Cauchy-Riemann geometry, making connections with physics, and present some recent results.